

Unifying all classical Force Equations

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Abstract :

It can be shown that all classical force equations can be derived from one another by means of a new definition of discrete electric and magnetic fields for localized massive particles [(5)], and that all of them amount to Newton's $F=ma$ fundamental acceleration equation.

It has long been established that all planets moving about the Sun follow elliptical orbits having the Sun at one focus (Kepler's first law) and that a line joining any planet to the Sun sweeps equal areas in equal times (Kepler's second law). Kepler also established that the square of the period (T) of any planet about the Sun is proportional to the cube of the planet mean distance (r) to the Sun (his third law). These laws however were only descriptive and offered no theoretical explanation as to the cause of such regularities.

It was Newton who later introduced the concept of force and confirmed the general soundness of his classical gravitational theory by deriving Kepler's three laws from his gravitational equations. Georges Gamow, Nobel prize winner for his contribution to relativistic theory, clearly summarizes how he proceeded in his popularization work "Gravity" ([1], Chapters 2, 3 and 4). Note that we find a similar although much less complete demonstration of Kepler's third law in Halliday and Resnick "Physics" [2, p 402].

Presently, it is obvious from analyzing Kepler's first and second laws that the motion of any planet about the Sun can mathematically be simplified at the limit as if it was circular at a distance from the Sun equal to the mean radius of the elliptical orbit. This is what allowed Newton to associate the centripetal acceleration of a circular motion (v^2/r) to orbital motion, where v is the velocity of a body (m) and r is the radius of its theoretical circular orbit.

$$F = ma = m \frac{v^2}{r} \quad (1)$$

Newton's basic postulate was that each planet and the Sun must be attracted to each other with a force proportional to the product of their masses and inversely proportional to the square of the distance separating them, a relation that mathematically can be represented by equation

$$F = G \frac{Mm}{r^2} \quad (2)$$

where M represents the mass of the Sun, m the mass of a planet and r the mean radius of this planet's orbit, G being a constant that was to be experimentally determined. Newton's insight here, as explained by Gamow, was that the centripetal acceleration multiplied by the mass of a planet should be equal to the gravitational force of attraction, hence from (1) and (2) :

UNIFYING CLASSICAL FORCE EQUATIONS

$$F = ma = \frac{mv^2}{r} = \frac{GMm}{r^2} \quad (3)$$

On the other hand, given that the length of a circular orbit is $2\pi r$, the period (T) of one revolution will be given by

$$T = \frac{2\pi r}{v} \quad \text{hence} \quad v = \frac{2\pi r}{T} \quad (4)$$

Substituting (4) in equation (3), we obtain

$$\frac{m}{r} \left(\frac{4\pi^2 r^2}{T^2} \right) = \frac{GMm}{r^2} \quad \text{and simplifying : } 4\pi^2 r^3 = GMT^2 \quad (5)$$

which clearly establishes, as Newton demonstrated, that the cube of the mean radius (r) of an orbit is proportional to the square of the orbiting body's period (T), which is very precisely Kepler's third law. Infinitesimal calculation would also show that the same law applies to elliptical orbits.

But equation (5) allows much more than only confirming Kepler's third law. It actually allows calculating G from the well known set of parameters of the Earth orbit, which will allow us to confirm the experimentally determined value of gravitational constant G! Strangely, no trace of such confirming calculation can be found in the literature! So, isolating G in equation (5), we obtain the following equation

$$G = \frac{4\pi^2 r^3}{MT^2} \quad (6)$$

The latest values of Earth orbit parameters as obtained from the **CRC Handbook of Chemistry and Physics** are M representing the estimated mass of the Sun ($M=1.9891E30$ kg), r representing the mean radius of the Earth orbit ($r=1.4959787E11$ m) and T representing the time for the Earth to complete one orbit, which is one year ($T=3.15581E7$ s). The reader can do the calculation himself for verification. The value of the Gravitational Constant having been experimentally established by various means as $G=6.673$ E-11 Newton m^2/kg^2 , let's see what value we mathematically obtain from Earth orbit parameters :

$$G = \frac{4\pi^2 r^3}{MT^2} = \frac{4\pi^2 (1.4959787E11)^3}{1.9891E30 \times (3.15581E7)^2} = 6.672024824E-11 \text{ N} \cdot \text{m}^2 / \text{kg}^2 \quad (7)$$

We observe that the value mathematically calculated for G is of course very close to the experimentally obtained value, since the experimental error margin is rated at 0.003 E-11 Nm^2/kg^2 .

Let us now examine another well known force equation, that allows calculation of the force at the ground state orbit of the Bohr atom, that is the Coulomb equation as applied to the isolated hydrogen atom, where k is the Coulomb constant, that resolves to $1/4\pi\epsilon_0$ in which ϵ_0 is the electrostatic permittivity constant of vacuum

$$F = k \frac{e^2}{r^2} = \frac{e^2}{4\pi\epsilon_0 r^2} = 8.238721806E-8 \text{ N} \quad (8)$$

Why refer here to the Bohr atom? Simply because it is commonly used in numerous textbooks to compare the electrostatic force (the Coulomb force) to the Gravitational force, and by this means, "prove" that the gravitational force is immensely weaker than the electrostatic force.

It is customary in introductory physics textbooks, for example the well renowned "Physics" by Halliday and Resnick [(1), p 1192] and so many others, to equate this equation with Newton's basic classical mechanics force equation $F=ma$ already mentioned as equation (1) in this paper as also being equated to the gravitational equation to prove Kepler's third law, to demonstrate that $F=ma$ gives the very same force as the Coulomb electrostatic force equation.

Using the known mass of the electron ($m=9.10938188E-31$ kg), the classical radius of the electron Bohr ground state orbit ($r=5.291772083E-11$ m), and the classical velocity of the electron on that Bohr ground state orbit ($v=2187691.253$ m/s), let us replay here this very well documented calculation.

$$F = \frac{e^2}{4\pi \epsilon_0 r^2} = ma = m \frac{v^2}{r} = 9.10938188 \text{ E-31} \frac{(2187691.253)^2}{5.291772083 \text{ E-11}} = 8.238721809 \text{ E-8 N} \quad (9)$$

And we effectively observe that the force calculated is exactly the same as with the Coulomb equation.

However, quite a few textbooks [(3), p 465], routinely give the following example to demonstrate that the electrostatic force (from the Coulomb equation) is immensely more intense than the gravitational force. From the two force equations

$$F_e = \frac{e^2}{4\pi \epsilon_0 r^2} \quad \text{and} \quad F_g = G \frac{Mm}{r^2} \quad (10)$$

where M is deemed to represent the mass of the proton and m the mass of the electron, a ratio is established that apparently reveals that the gravitational force is 39 orders of magnitude less intense than the electrostatic force :

$$\frac{F_e}{F_g} = \frac{e^2}{4\pi \epsilon_0 GMm} = 2.269 \text{ E39} \quad (11)$$

How can this be, since we just verified that both the gravitational equation (3) and the electrostatic equation (9) are verifiably and traditionally equated to Newton's basic force equation $F=ma$ to demonstrate their validity? We just saw how equations (3) to (6) show how Newton demonstrated that his gravitational equation allows deriving Kepler's third law when equated to $F=ma$, and we just saw how $F=ma$ is used in textbooks to calculate the same force as the Coulomb force equation when applied to the classical Bohr ground state orbit in the hydrogen atom.

Reexamining equation (7) will give us the key to this apparent paradox. We observe that G , the so-called universal gravitational constant, in the traditional form allowing deriving Kepler's third law, makes use of 3 variables whose sizes, although appropriate for astronomical purposes, are way out of range for dealing with atomic scale values, which is precisely what is traditionally being done with ratio (11), which led to the apparently faulty comparison that can be found in so many textbooks.

We observe that using standard G to calculate the force inside an atom just about amounts to calculating the energy of an electron orbiting the Sun at Earth orbit, since the actual estimated mass of the Sun, as well as the radius or the Earth orbit and the time it takes for the Earth to complete one revolution are directly embedded into G and can hardly counterbalance the mass of the proton and the time for one electron revolution in the Bohr atom.

Considering equation (7) again, it seems rather simple to correct this problem by using values for central mass, orbit radius and time for one Bohr electron orbit that are coherent with the atomic scale to which the Coulomb equation applies.

Let us first establish the values that we need to calculate a G specific to the hydrogen atom. First, we have the known Bohr radius $r_0 = 5.291772083E-11$ m. From the known frequency (6.57968391E15 Hz) of the energy (27.21138345 eV) induced at Bohr's radius by the Coulomb force, we can calculate time T taken for one orbit of the electron in the Bohr model:

$$T = 1 \text{ sec} / 6.57968391E15 \text{ Hz} = 1.519829851E-16 \text{ sec.} \quad (12)$$

The effective mass of the proton being rated as $M = 1.67262158E-27$ kg, let us now calculate a value of G that applies to the effective mass of the proton :

$$G_p = \frac{4\pi^2 r_0^3}{M_p T^2} = 1.514172983E29 \text{ N} \cdot \text{m}^2 / \text{kg}^2 \quad (13)$$

Let's now recalculate the force at the Bohr radius from the gravitational equation :

$$F_g = G_p \frac{M_p m_e}{r_o^2} = 8.238721759 \text{ E} - 8 \text{ N} \quad (14)$$

So, we observe that the so-called "universal gravitational constant" G, may not be as universal as is generally believed! We just verified, contrary to the obviously faulty demonstration made in numerous textbooks, that we now obtain the very same force with the gravitational equation (14) that can be calculated with the Coulomb equation (8), and if we recalculate ratio (11) with this correctly amended value of G, we finally obtain 1 as a result, **meaning that both forces can only be identical.**

$$\frac{F_c}{F_g} = \frac{e^2}{4\pi \epsilon_0 G_p M_p m_e} = 1 \quad (15)$$

Now, if we resolve G to its detailed definition (13) for calculating the force in the Gravitational equation (14), we immediately notice that M cancels out if both occurrences of M coherently represents the central mass of the orbital system (here the mass of the proton) being considered and that r simplifies considerably if the r built into the definition of G is the same as the r from the gravitational force equation, that is, in this case, the radius of the Bohr orbit.

$$F = \frac{4\pi^2 r_o^3}{M_p T^2} \frac{M_p \bullet m_e}{r_o^2} \quad (16)$$

$$F = \frac{4\pi^2 r_o m_e}{T^2} = 8.238721759 \text{ E} - 8 \text{ N} \quad (17)$$

Interestingly, as we now observe, it seems that to calculate the force acting between an orbiting body and a central mass, we don't even need the central mass of the system¹, but only the

¹ As a side issue, this sheds an entirely new light on the manner in which the gravitational constant has been used since Cavendish to supposedly "measure" the masses of Solar System bodies. The fact that the orbiting body mass simplifies out of the definition of G as revealed by Gamow ([1],p.62) and that the central mass simplifies out of the final force equation reveals that the most information we really obtain from the gravitational equation applied to the Solar System are mass ratios.

To really measure the masses of Solar System bodies, there is need to compare at least one naturally orbiting Solar system mass with a man made orbiting body whose mass will have been measured before launch at ground level.

The way to do it is to send the largest payload possible to orbit the smallest naturally orbiting satellite in the solar system that we can easily observe (Phobos or Déimos, or one of Jupiter's satellites, maybe), measure the wobble of the satellite against the translating mass of the payload, which will then allow really measuring the satellite mass, then the mass of its primary, and then from the very precise wobble ratios that we have for all other solar system bodies, their respective masses, and finally, the true mass of the Sun and Earth.

Here is how this wobble is to be calculated. Although the gravitational equation gives the correct attractive force between two orbiting bodies, it does not provide the center of rotation about which the two bodies orbit. Let's see how we can find that center of rotation. A very simple rule of basic mechanics reveals that in a system of two captive bodies in translation about a common center, the product of the radius of translation of one mass and of that mass will be equal to the product of the radius of translation of the other mass by that other mass

$$Mx = m(r-x)$$

We can now redefine m_e in terms of M_p , a_o and x .

$$m = \frac{M_p x}{a_o - x}$$

We can now substitute the value of m in the gravitational force equation

$$F = \frac{G_{p2} M_p M_p x}{a_o^2 (a_o - x)} = \frac{G_{p2} M_p^2 x}{a_o^3 x - a_o^2 x}$$

If we isolate x, and calculate, we obtain

$$x = \frac{F a_o^3}{G_{p2} M_p^2 + F a_o^2} = 2.880420459 \text{ E} - 14 \text{ m}$$

length of the orbit ($\lambda=2\pi r$) and the time taken to complete one orbit, that is, the inverse of the frequency ($1/f$).

Traditionally, the gravitational equation is "equated" to the fundamental force equation ($F=ma$) to derive Kepler's third law, but we will soon prove also, that the gravitational equation itself ultimately reduces to the simplest fundamental force equation $F=ma$, but ONLY if the central mass M , radius r and time T defining G (equations (7) and (13)) are consistent with the M and r values used in the complete gravitational equation (equations (2) and (14)).

Let us now multiply both sides of equation (17) by r to obtain the energy induced at the distance separating the two bodies in equation (14)

$$E = Fr = \frac{4\pi^2 r_o^2 m_e}{T^2} = \frac{(2\pi r)^2 m}{T^2} = (\lambda f)^2 m = 4.359743805 \text{ E-18 J} \quad (18)$$

which we verify still yields the correct energy induced at the Bohr orbit, just like the Coulomb equation.

$$E = Fr = \frac{e^2}{4\pi \epsilon_0 r^2} = 4.359743805 \text{ E-18 J} \quad (19)$$

We saw with equations (16) and (17) that once the variables making up the definition of G are integrated into gravitational equation, the equation reduces to.

$$F = \frac{4\pi^2 r m}{T^2} \quad \text{which we can reorganize as} \quad F = mr \left(\frac{2\pi}{T} \right)^2 \quad (20)$$

Multiplying and dividing equation (20) by mutually canceling occurrences of r allows the following transformation

$$F = mr \left(\frac{2\pi}{T} \right)^2 \frac{r}{r} = \frac{m}{r} \left(\frac{2\pi r}{T} \right)^2 \quad (21)$$

Since the length of an orbit ($2\pi r$) divided by the time taken to complete it (T) is the velocity of the orbiting body (v), we can then write

$$F = \frac{m}{r} \left(\frac{2\pi r}{T} \right)^2 = \frac{m}{r} (v)^2 = m \frac{v^2}{r} = ma \quad (22)$$

Now, after having demonstrated that the gravitational equation reduces to $F=ma$, what if we similarly demonstrated that the Coulomb equation also reduces to $F=ma$ for the electron at the Bohr orbit!

By substituting in (8) a little documented but standard definition of the electrostatic permittivity constant of vacuum ($\epsilon_0=1/(4\pi c^2 \cdot 10^{-7})$), the coulomb equation can be reformulated as follows

$$F = \frac{1}{4\pi \epsilon_0} \frac{e^2}{r_o^2} = \frac{4\pi c^2 \cdot 10^{-7}}{4\pi} \frac{e^2}{r_o^2} = \frac{e^2 \cdot 10^{-7}}{r_o^2} \frac{c^2}{r_o^2} \quad (23)$$

From a development regarding the magnetic field of a moving electron published by Paul Marmet in 2003 [(4)], a new and very useful definition of energy was derived in a previous paper

which is the radius of the orbit that the proton traces about the common center of translation that it shares with the electron, and is a reflection of the wobble of the proton in the hydrogen atom due to the motion of the electron on its orbit, as hypothesized by Bohr. So, let's confirm by recalculating the mass of the electron with the equation for the center of translation of the system.

$$m = \frac{M_p x}{a_0 - x} = 9.10938194 \text{ E-31 kg}$$

which is exactly the well known mass of the electron.

[(5), equation (11)] involving fundamental constants and the absolute wavelength of the energy considered

$$E = hf = \frac{e^2}{2\epsilon_0\alpha\lambda} \quad (24)$$

Of course, if we use the electron Compton wavelength (λ_c) in equation (24) and divide this energy by the square of the speed of light, we will obtain a corresponding new definition of the electron mass.

$$m_e = \frac{E}{c^2} = \frac{e^2}{2\epsilon_0\alpha\lambda_c c^2} \quad (25)$$

Now, if we operate in (25) the same substitution for the electrostatic permittivity constant of vacuum (ϵ_0) that we carried out in equation (23), we obtain the following new definition of the electron rest mass

$$m_e = \frac{e^2}{2\epsilon_0\alpha\lambda_c c^2} = \frac{4\pi c^2 \cdot 10^{-7} \cdot e^2}{2\alpha\lambda_c c^2} = \frac{2\pi \cdot 10^{-7} \cdot e^2}{\alpha\lambda_c} = \frac{e^2 \cdot 10^{-7}}{\lambda_c} \frac{2\pi}{\alpha} \quad (26)$$

If we compare this new definition of the electron mass with equation (23), we observe that if we wanted to use definition (26) in equation (23), we would have to multiply equation (23) by mutually canceling integrated amplitudes of the electron's energy ($\lambda_c\alpha/2\pi$). So from (23)

$$F = \frac{e^2 \cdot 10^{-7}}{r_o^2} \frac{c^2}{\alpha} = \left(\frac{e^2 \cdot 10^{-7}}{\lambda_c} \frac{2\pi}{\alpha} \right) \frac{\lambda_c \alpha}{2\pi} \frac{c^2}{r_o^2} \quad (27)$$

We can now observe that the expression between parentheses in equation (27) is identical to the new definition of the electron mass that we derived in equation (26). So let's replace in (27) the expression in parentheses by the usual symbol of the electron rest mass

$$F = \left(\frac{e^2 \cdot 10^{-7}}{\lambda_c} \frac{2\pi}{\alpha} \right) \frac{\lambda_c \alpha}{2\pi} \frac{c^2}{r_o^2} = m_e \frac{\lambda_c \alpha}{2\pi} \frac{c^2}{r_o^2} \quad (28)$$

Now we know that the theoretical velocity of the electron at the Bohr orbit is equal to the speed of light multiplied by the fine structure constant $v=\alpha c$. Since the speed of light is squared in equation (28) and that α is not squared, we need to multiply and divide the equation by mutually canceling occurrences of α so we can convert the squared speed of light to squared theoretical velocity of the electron on the Bohr orbit (classical velocity $v=2,187,691.252$ m/s). So, let's proceed from (28)

$$F = m_e \frac{\lambda_c \alpha}{2\pi} \frac{c^2}{r_o^2} = m_e \frac{\lambda_c}{2\pi \alpha} \frac{\alpha^2 c^2}{r_o^2} = m_e \frac{\lambda_c}{2\pi \alpha} \frac{v^2}{r_o^2} \quad (29)$$

Finally, a little calculation on a pocket calculator will confirm that $\lambda_c/2\pi\alpha$ restitutes very precisely the Bohr radius (r_0). So we can carry out the proper substitution in equation (29) and finally obtain $F=ma$ as initially intended

$$F = m_e \frac{\lambda_c}{2\pi \alpha} \frac{v^2}{r_o^2} = m_e \frac{r_o}{r_o} \frac{v^2}{r_o^2} = m_e \frac{v^2}{r_o} = m_e a \quad (30)$$

So, we have here the mathematical proof that gravitation does apply within atoms when using the proper values just as it does in the Solar System, which is demonstrated by successfully reducing both the gravitational equation and the coulomb equation to the same fundamental acceleration equation $F=ma$ that is traditionally used to prove on one hand that conformity of the gravitational equation with Kepler's third law, and on the other hand, to prove that the Coulomb equation is in agreement with classical mechanics.

We have now demonstrated the complete fundamental identity of the following three classical equations

$$F = G \frac{Mm}{r^2} = k \frac{e^2}{r^2} = ma \quad (31)$$

But there is more! We can also derive $F=ma$ from the Lorentz force equations calculated from the electric and magnetic fields of the Bohr ground state energy $F=evB$ and $F=e\alpha E$.

From the definition of the magnetic field derived from the Marmet paper in a previous paper on discrete fields [(5), equation (34)], we can write

$$F = evB = ev \frac{\mu_0 \pi ec}{\alpha^3 \lambda^2} \quad (32)$$

In a procedure similar to that used for equation (23), we will now replace μ_0 by its π -related definition ($\mu_0=4\pi 10^{-7}$)

$$F = ev \frac{\mu_0 \pi ec}{\alpha^3 \lambda^2} = \frac{e^2 10^{-7}}{\alpha^3 \lambda^2} \frac{4\pi^2 vc}{\alpha} \quad (33)$$

We previously determined in equation (26) that the rest mass of an electron can be represented by the following equation

$$m_e = \left(\frac{e^2 10^{-7}}{\lambda_c \alpha} \right) \quad (34)$$

where λ_c is the electron Compton wavelength.

On the other hand, since the above force equation (33) applies to the Bohr ground state energy, the energy involved is thus $4.359743805E-18$ J with an absolute wavelength of $4.556335254E-8$ m. A little calculation will show that when this energy is multiplied by the square of the fine structure constant (α^2), we recuperate the electron Compton wavelength ($\lambda_c=\lambda\alpha^2$). So, let's substitute this value in force equation (33)

$$F = \frac{e^2 10^{-7}}{(\lambda \alpha^2) \alpha \lambda} \frac{4\pi^2 vc}{\lambda_c \alpha} = \frac{e^2 10^{-7}}{\lambda_c \alpha \lambda} \frac{4\pi^2 vc}{\lambda_c \alpha} = \left(\frac{e^2 10^{-7}}{\lambda_c \alpha} \right) \frac{2\pi vc}{\lambda} \quad (35)$$

Comparing now the force equation (35) with the mass equation (34), we observe that the mass equation presently is a subset of the force equation. So, let's replace that subset with the rest mass symbol of the electron

$$F = \left(\frac{e^2 10^{-7}}{\lambda_c \alpha} \right) \frac{2\pi vc}{\lambda} = m_e \frac{2\pi vc}{\lambda} \quad (36)$$

Now, to obtain the Bohr radius from the absolute wavelength of the absolute Bohr ground state wavelength, one needs to multiply the amplitude of that wavelength by alpha (α) and to introduce that occurrence of alpha, we need to multiply and divide the equation by mutually reducible occurrences of alpha. So, let's proceed.

$$F = m_e \frac{2\pi v \alpha c}{\lambda \alpha} = m_e \frac{v \alpha c}{r_0} \quad (37)$$

Finally, it is easy to verify that the speed of light multiplied by the fine structure constant (αc) restitutes the classical velocity of the electron on the Bohr ground orbit. So, let's operate this last substitution

$$F = evB = m_e \frac{v \alpha c}{r_0} = m_e \frac{v^2}{r_0} = m_e a \quad (38)$$

which is the proof that the Lorentz magnetic force equation, just like the gravitational force equation and the Coulomb force equation, is just another form of the same fundamental Newton acceleration equation $F=ma$.

Let's now have a look at the last remaining classical force equation

$$F = e\alpha E \quad (39)$$

which is obtained from equality $F=eE=ecB$ found valid for localized photons [(6), chapter Carrier-Photons and the Intermediate Equation Set], in which the velocity is reduced to the Bohr ground state classical velocity (since $v_{\text{Bohr}} = \alpha c$) to involve the electron mass. So multiplying both members of the equation by α reduces the force associated to the electric field to that consistent with Bohr ground orbit parameters.

We determined in a previous paper fields [(5), equation (40)] that the definition of the electron magnetic field derived from the Marmet paper allows redefining the electric field as

$$E = \frac{\pi e}{\epsilon_0 \alpha^3 \lambda^2} \quad (40)$$

So, substituting for E in equation (39) we obtain

$$F = e\alpha E = \frac{e\alpha}{\epsilon_0 \alpha^3 \lambda^2} \frac{\pi e}{\lambda^2} \quad (41)$$

Reducing ϵ_0 to its internal components ($1/4\pi c^2 10^{-7}$), we obtain

$$F = \frac{e\alpha}{\epsilon_0 \alpha^3 \lambda^2} \frac{\pi e}{\lambda^2} = \frac{e^2 10^{-7} 4\pi^2 \alpha c^2}{\alpha^3 \lambda^2} \quad (42)$$

Since we already established that $\lambda_c = \lambda \alpha^2$, we can substitute as follows

$$F = \frac{e^2 10^{-7} 4\pi^2 \alpha c^2}{\alpha^3 \lambda^2} = \frac{e^2 10^{-7} 4\pi^2 \alpha c^2}{\lambda_c \alpha \lambda} \quad (43)$$

And we also established with equation (26) that

$$m_e = \left(\frac{e^2 10^{-7} 2\pi}{\lambda_c \alpha} \right) \quad (44)$$

so we can operate the following substitution

$$F = \frac{e^2 10^{-7} 4\pi^2 \alpha c^2}{\lambda_c \alpha \lambda} = \left(\frac{e^2 10^{-7} 2\pi}{\lambda_c \alpha} \right) \frac{2\pi \alpha c^2}{\lambda} = m_e \frac{2\pi \alpha c^2}{\lambda} \quad (45)$$

We also established that the Bohr radius (a_0) is equal to the absolute wavelength of the Bohr ground state energy multiplied by alpha (α), and to allow this reduction, we see that we need to multiply and divide the equation by mutually reducible occurrences of α . So, let's proceed

$$F = m_e \frac{2\pi \alpha c^2}{\lambda} = m_e \frac{2\pi}{\lambda \alpha} \frac{\alpha^2 c^2}{\alpha} = m_e \frac{\alpha^2 c^2}{a_0} \quad (46)$$

Finally, we know that multiplying the speed of light by α restitutes the classical Bohr ground state velocity, so we finally obtain

$$F = m_e \frac{\alpha^2 c^2}{a_0} = m_e \frac{v^2}{a_0} = m_e a \quad (47)$$

This completes the demonstration.

Let us take good note here that all of the classical force equations ultimately resolve as involving a mass being accelerated.

Conclusion

The very simple conclusion of this sequence of derivations is that only one force seems to be at play for all of these equations, including the gravitational equation.

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